E4-6$\mathcal{CG}$: Exploring Projective Conics[1]

By Judith N Cederberg

Introduction:

In projective geometry there are two types of conics, namely point and line conics. Once again, since the elements of these sets are defined strictly in terms of points and lines, the capabilities of dynamic geometry software greatly enhance the construction and exploration of these sets. The activities below are similar to, and extend, Exercises 1 and 2 in Section 4.6 of Cederberg. Exercises 5 and 6 in that same section are also most efficiently done using dynamic geometry software.

Equipment and Materials Needed:

1. A computer (Macintosh or a PC running Windows) that has access to the program *Cabri Geometry II*[2] and a disk on which to save files.

2. Figures "Hom-cen" and "Hom-axis" constructed in Exploration **E4-5** or the figures "Pen-lns" and "Pen-pts" used in **E4-5**.

In the first activity below you will find five points of the point conic determined by a projectivity between two pencils of lines. Then you will use a center of homology to find additional points of the point conic. Finally you will observe how the conic changes when you change a line in one of the pencils. Activities 4 and 5 explore line conics.

1. Constructing a point conic:

   (a) Reopen your figure "Hom-cen.fig" from Exploration **E4-5** or open the figure "Pen-lns.fig" and use the suggestions in activity 5 of that exploration to find the center of homology.

   (b) Use *Hide/Show* in the **Draw** toolbox to hide lines d, d', e, e', your written comment, and all the lines and points you used in your construction of H. Do **not** hide H.

   (c) Find and label the points $A = a \cdot a'$, $B = b \cdot b'$, and $C = c \cdot c'$. Be sure to use the *Intersection Point* in the **Points** toolbox. Note that these points as well as the points P and P' lie on the point conic determined by the projectivity $abc \wedge a'b'c'$[3].

   (d) To have Cabri draw the conic, select *Conic* in the **Curves** toolbox. Then select the 5 points, P, P', A, B, and C. Open a *comment window* and describe the shape that appears.

   (e) Save your figure as "Pt-con1.fig"

2. Finding additional points of the point conic:

   (a) In the same figure, construct a circle centered at P by selecting *Circle* in the **Curves** toolbox, selecting P and then dragging the cursor to an arbitrary point to determine the radius of the circle.

   (b) Using *Point on Object* (in the **Points** toolbox), find and label a point X on the circle, but not on lines a, b or c.

   (c) Construct and label the line $f = PX$.

   (d) Use the center of homology H to find f', the ray corresponding to f under the projectivity, and construct the point $F = f \cdot f'$, another point of the point conic. Note: You may need to move X around the circle to obtain a convenient position for f.

   (e) Turn on the *Trace* in the **Display** toolbox and select point F.

   (f) Use the *pointer* to drag X around your circle and observe the locus of point F.

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[1] Designed to supplement Section 4.6 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC July 18, 2000

[2] Trademark of Université Joseph Fourier, distributed in the US by Texas Instruments Inc.

[3] See Definition 4.11 and Theorem 4.12 in section 4.6 of Cederberg.

(g) Open a *comment window* and describe what happened.

(h) Save your construction as "Pt-con2.fig"

3. Changing the projectivity defining the point conic:

   (a) Working with the same construction, eliminate the locus by selecting *Refresh Drawing* under the **Edit** menu. Then turn off the *Trace* by clicking in free space while pressing the *Shift* key.

   (b) Turn on *Show Axis* in the **Draw** toolbox.

   (c) Turn on the *Equation* feature in the **Measure** toolbox and select your conic. Use the *pointer* to move the equation to a convenient position.

   (d) Change the color of line a so you can easily identify it.

   (e) Make your conic thick by choosing *Thick* in the **Draw** toolbox.

   (f) Now drag line a, one of the lines defining the projectivity, around P. Watch what happens to the shape and equation of your conic.

   (g) Drag line a so that the resultant conic appears to be a Euclidean ellipse. Then repeat parts (e) and (f) in Activity 2.

   (h) In your *comment window* summarize what happens when you change the position of line a.

   (i) Save your construction as "Pt-con3.fig"

4. Constructing a line conic:

   (a) Reopen your figure "Hom-axis.fig" from Exploration **E4-5** or open the figure "Pen-pts.fig" and use the suggestions in activity 6 of that exploration to find the axis of homology.

   (b) Hide E, E', and all the lines and points you used in your construction of D', E' and h. Do **not** hide h.

   (c) Construct lines AA', BB' and CC' and DD'. Color these light blue so that they are the same color as the lines $p = AB$ and $p' = A'B'$. Note that these light blue lines are lines of the line conic determined by the projectivity $ABC \wedge A'B'C'$[4].

   (d) Highlight line DD' by making it a thick line.

   (e) Turn on *Trace* in the **Display** toolbox and then select line DD'.

   (f) Use the pointer to drag D along line p. Be sure to drag D as far as possible both to the right and to the left. What shape appears to be "enclosed" by the locus of the line DD'?

   (g) Open a *comment window* and describe your observations.

   (h) Save your construction as "Ln-con1.fig"

5. Changing the projectivity defining the line conic:

   (a) Working with the same construction, eliminate the locus by selecting *Refresh Drawing* under the **Edit** menu. Then turn off the *Trace* by clicking in free space while pressing the *Shift* key.

   (b) Using the *Pointer*, move line p' so that it intersects p between points A and B. Be sure you can still see point D.

   (c) Repeat parts (d)–(f) in Activity 4 above.

   (d) Change your description in the comment window to reflect your current observations.

   (e) Save your construction as "Ln-con2.fig"

Report:

Submit a computer folder titled "E4-6CG." This folder should contain the constructions listed below and any other dynamic geometry software figures you made for the activities in section 4.6 of the text *A Course in Modern Geometries, 2nd. Ed.*[5]

(a) Pt-con1.fig   (b) Pt-con2.fig   (c) Pt-con3.fig   (d) Ln-con1.fig   (e) Ln-con2.fig

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[4]See Definition 4.13 and the dual of Theorem 4.12 in section 4.6 of Cederberg.

[5]For each of these others, include a comment/caption describing the purpose of the construction.