E3-4$\mathcal{DGS}$: Exploring Translations & Frieze Pattern Symmetries[1]

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Introduction:

The translation and glide reflection transformations can be implemented using dynamic geometry software. The effects of these transformations are best observed by applying them to a nonsymmetric figure such as a flag.[2] The directions below indicate how to implement translations and glide reflections using both *Cabri Geometry II (*$\mathcal{CG}$*)* and *Geometer's Sketchpad (*$\mathcal{GS}$*)*. With these directions, you can carry out the activities in Section 3.4 of *A Course in Modern Geometry, 2nd Ed.*

Equipment and Materials Needed:

A computer (Macintosh or a PC running Windows) with access to either *Cabri Geometry II* or *Geometer's Sketchpad (Version 3 or 4)*, and a disk on which to save files.

3.4.1 Implementing Translations:

$\mathcal{GS}$ *Translation of an object F along vector $\overrightarrow{AB}$:* $(T_{\vec{AB}}(F))$

Construct and label two points A and B. Select points A and B and choose *Mark Vector* from the **Transform Menu.** Then select object F and choose *Translate* in the **Transform Menu**.

$\mathcal{CG}$ *Translation of an object F along vector $\overrightarrow{AB}$:* $(T_{\vec{AB}}(F))$

Using *Vector* from the **Lines Toolbox**, construct vector $\overrightarrow{AB}$, labeling the endpoints as you draw the vector. Then using *Translation* from the **Transform Toolbox**, click on object F and $\overrightarrow{AB}$.

3.4.2 Implementing Glide Reflections:

Neither *Geometer's Sketchpad* or *Cabri Geometry* have a "built-in" glide reflection. In both cases, you can implement glide reflections by sequentially performing a translation and reflection that produce the desired glide reflection.

3.4.3 Generating Frieze Patterns:

With the instructions above, you can use dynamic geometry software to carry out the activities in Section 3.4 of the text. In addition, you can use this software to easily create an example of each of the seven possible frieze patterns. The first activity below leads you through one construction.

1. Using the instructions below construct a frieze pattern with a half-turn, i.e., a point reflection, and a translation.

   $\mathcal{CG}$ *Applying a half-turn, H_C:* Use the *Symmetry Tool* in the **Transform Toolbox**.

   $\mathcal{GS}$ *Applying a half-turn, H_C:* Use the equivalent rotation $R_{C,180}$.

   (a) Construct a line c and hide the two points defining c. Construct and label a point C on c to serve as the center of your half-turn H_C. Construct a second point C' also on line c so that $\overrightarrow{CC'} = \mathcal{T}$; i.e., $\mathcal{T}$ is the vector of the generating translation.

   (b) Construct and fill a nonregular polygon $\mathcal{P}$.[3]

   (c) Then repeatedly apply to $\mathcal{P}$ both the half-turn H_C and the translation $\mathcal{T}$ to generate a portion of a frieze pattern $\mathcal{F}_c$. Note that you may want to make use of either the $\mathcal{CG}$ Macro or the $\mathcal{GS}$ Custom Tool/Script feature to carry out the successive applications of these transformations.

   (d) How many points of symmetry does your frieze pattern $\mathcal{F}_c$ have? How can you find them?

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[1] Designed to supplement Section 3.4 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC July 24, 2000

[2] For instructions on making such figures with *Cabri Geometry II* and *Geometer's Sketchpad* see E3-2$\mathcal{DGS}$.

[3] For instructions on making a nonregular polygon with *Cabri Geometry II* and *Geometer's Sketchpad* see E3-2$\mathcal{DGS}$.

(e) Does your frieze pattern $\mathcal{F}_c$ have any symmetries in addition to translational and point symmetries, e.g., line or glide symmetry? If so, indicate the lines (or axes) of each type of symmetry.

(f) Save your construction and comments/captions answering the previous questions as "$Fc - Tr - Ht$."

2. For each type of symmetry listed below, construct a portion of a frieze pattern that has only the named symmetries in addition to translational symmetry.[4] Clearly label your center c, the translation vector $\mathcal{T}$ and several of the points or lines of symmetry. For each frieze pattern, give a caption/comment to describe the transformations you used to generate the pattern and save it using the suggested title in parentheses.

   (a) No other symmetry (Fc-none)

   (b) Point symmetry (Fc-point)

   (c) Glide symmetry (Fc-glide)

   (d) Perpendicular line symmetry (Fc-perp)

   (e) Center line and glide symmetries (Fc-cl-gl)

   (f) Glide and perpendicular line symmetries (Fc-gl-pr)

   (g) Center line, perpendicular line, point and glide symmetries (Fc-all)

Report:

Submit a computer folder titled "E3-4CG" or "E3-4GS" (depending on whether you used *Cabri* or *Sketchpad*). This folder should contain any dynamic geometry software figures/sketches you made for the activities in section 3.4 of the text *A Course in Modern Geometries, 2nd. Ed.*[5] and the frieze patterns listed above. For each figure/sketch, be sure to use the appropriate extension, i.e., ".gsp" for *Sketchpad* sketches and ".fig" for *Cabri* figures.

[4] Note that the frieze pattern created above is an example of one of these frieze patterns.

[5] For each of these others, include a caption describing the purpose of the construction.