E3-3,4 $\mathcal{CG}$: Exploring Symmetries with Cabri[1]

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Introduction:

The transformations known as symmetries can be implemented using dynamic geometry software. The effect of a symmetry is best observed by applying it to a nonsymmetric object such as a flag. The directions below indicate how to implement symmetries using *Cabri Geometry II (CG)*. Following these directions, there are activities similar to some of those found in Sections 3.3 and 3.4 of *A Course in Modern Geometry, 2nd Ed.*. Other activities in that section can also be implemented using either *Cabri* or *Sketchpad*.

3.3.1 Reflections, Rotations, and Translations on Cabri

- *Reflecting an object F in axis m: ($R_m(F)$)*

  In the **Transform Toolbox** select *Reflection*, and click successively on m and F.

- *Rotating an object F by θ^0 about center C*: ($R_{C,\theta}(F)$)

  In the **Display Toolbox** select *Numerical Edit* and type the value of θ in the box that appears. To record the units, press the *CTRL* and *U* keys to display the popup unit menu. Now in the **Transform Toolbox** select *Rotation*, and click successively on F, C and the angle value.

- *Translating an object F along vector $\overrightarrow{AB}$*: ($T_{\overrightarrow{AB}}(F)$)

  In the **Transform Toolbox** select *Translation*, and click successively on vector $\overrightarrow{AB}$ and F.

3.3.2 Rotation and Reflection Activities:

Using Cabri to implement symmetries as described above, complete each of the following:

1. Construct and label a small flag (or nonregular polygon) F_0 and a point P (not on the flag). Note: To get a flag that Cabri recognizes as one object, use the polygon (not regular polygon) tool.

    (a) Rotate flag F_0 by 120^0 about center P.
    (b) Label your new flag F_1.
    (c) Then perform the same rotation on F_1, etc. Do this until you end up with a rotated flag coinciding with the original flag F_0.
    (d) Create a caption/comment that summarizes your observations. In particular, answer the following questions:
        i. Which symmetries (i.e., line or rotational) are exhibited by the "multi-flag" construction that you created?
        ii. Why can your multi-flag construction be said to have *3-fold rotational symmetry*?
        iii. Save (but do not close) your completed construction as "Rot-Flag."

2. In your previous construction, add (and label)a point $Q \neq P$ and a line ℓ through P and Q. Rotate ℓ by 60° about P to get line m, so that $\angle(\ell, m) = 60^o$.

    (a) Reflect F_0 in line ℓ to obtain the image flag. Change the color of this new flag and label it F_a, i.e., $F_a = R_\ell(F_0)$.
    (b) Then find and label $F_b = R_m(F_a)$ and $F_c = R_\ell(F_b)$.
    (c) Continue this reflection process, first reflecting in line ℓ and then in line m until you end up with a flag on top of F_0.

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[1]Designed to supplement Sections 3.3 & 3.4 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC Feb. 5, 2007

(d) Now using the *rotate tool* select point Q and drag it to rotate line ℓ about P .(The angle $\angle(\ell, m)$ should remain the same during this rotation.)

(e) Replace your previous caption/comment with a new version that summarizes your observations. In particular, answer the following questions:

  i. Which symmetries are exhibited by your "multi-flag" figure?

  ii. How is the multi-flag figure generated by these reflections related to the figure generated previously by your rotations?

  iii. Describe how the image of a flag under a rotation can also be obtained by using only reflections. Be sure to explain how many reflections are needed and describe the relation of the axes, i.e., mirror lines, to the center and angle of the rotation.

  iv. When the location of line ℓ (and consequently of m) is changed without changing the intersection point or angle between these lines, which flag locations change? Which stay the same?

  v. How many different pairs of lines ℓ and m can be used as axes of reflections so that the result of reflecting Flag F_0 in ℓ and then in m is the same as the result of rotation Flag F_0 around center P through angle θ?

(f) Save your final construction as "R&R-Flag."

3.3.3 Translation and Reflection Activities:

1. Open the figure Flag Pair 1 found in the L Class Materials folder.

  (a) Find and label a vector $\overrightarrow{AB}$ such that the translation $T_{\overrightarrow{AB}}(F_0) = F_1$

  (b) Try to find two lines ℓ and m such that $R_m R_l(F_0) = F_1$. (Hint: Should your lines ℓ and m be parallel or intersect?) Use a text box to describe your answer to the question: How are the lines ℓ and m related to the vector $\overrightarrow{AB}$?

  (c) Save your figure as Flag-Pair-1****.

2. Open the figure Flag Pair 2 found in the L Class Materials folder.

  (a) Find a translation and a reflection that will map F_0 onto F_1. Label both the vector and axis you use.

  (b) Challenge: Find and label a single vector $\overrightarrow{AB}$ so that a reflection across AB followed by the translation $T_{\overrightarrow{AB}}$ will map F_0 to F_1.

  (c) Use a text box to explain your process.

  (d) Save your figure as Flag-Pair-2**

Report:

Submit a computer folder titled "E3-3CG." This folder should contain the figures listed below.

  (a) Rot-Flag   (b) R&R-Flag   (c) Flag-Pair-1   Flag-Pair-2