E3-3$\mathcal{DGS}$: Exploring Rotations and Finite Symmetry Groups[1]

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Introduction:

The transformations known as rotations can be implemented using dynamic geometry software. The effect of a rotation, like that of a reflection, is best observed by applying it to a nonsymmetric object such as a flag.[2] The directions below indicate how to implement rotations using both *Cabri Geometry II ($\mathcal{CG}$)* and *Geometer's Sketchpad ($\mathcal{GS}$)*. Following these directions, there are directions for implementing activities similar to those in the first activity set in Section 3.3 of *A Course in Modern Geometry, 2nd Ed.*. Other activities in that section can also be implemented using either *Cabri* or *Sketchpad.*

Equipment and Materials Needed:

A computer (Macintosh or a PC running Windows) with access to either *Cabri Geometry II* or *Geometer's Sketchpad (Version 3 or 4)*, and a disk on which to save files.

3.3.1 Implementing Rotations

$\mathcal{GS}$ *Rotation of an object F by θ^0 about center C: $(R_{C,\theta}(F))$*

Select point C and under the **Transform Menu** choose *Mark Center*. Now select F, and under the **Transform Menu** choose *Rotate*. Enter the value of θ for your angle and click on *OK*.

$\mathcal{CG}$ *Rotation of an object F by θ^0 about center C: $(R_{C,\theta}(F))$*

In the **Display Toolbox** select *Numerical Edit* and type the value of θ in the box that appears. To record the units, press the *CTRL* and *U* keys to display the popup unit menu. Now in the **Transform Toolbox** select *Rotation*, and click successively on F, C and the angle value.

3.3.2 Rotation Activities:

The activities below essentially duplicate those given in Subsection 3.3.1 of the text.

1. Construct and label a small flag (or nonregular polygon) F_0 and a point P (not on the flag).
   (a) Rotate flag F_0 by 120^0 about center P.
   (b) Label your new flag F_1.
   (c) Then perform the same rotation on F_1, etc. Do this until you end up with a rotated flag coinciding with the original flag F_0.
   (d) Create a caption/comment that summarizes your observations. In particular, answer the following questions:
      i. Which symmetries (i.e., line, point, rotational) are exhibited by the "multi-flag" construction that you created?
      ii. Why can your multi-flag construction be said to have *3-fold rotational symmetry*?
   (e) Save (but do not close) your completed construction as "Rot-Flag."
2. In your previous construction, add (and label) a line ℓ through P. Rotate ℓ by 60^o about P to get line m, so that $\angle(\ell, m) = 60^o$.
   (a) Reflect F_0 in line ℓ to obtain the image flag. Change the color of this new flag and label it F_a, i.e., $F_a = R_\ell(F_0)$.
   (b) Then find and label $F_b = R_m(F_a)$ and $F_c = R_\ell(F_b)$.

[1] Designed to supplement Section 3.3 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC July 8, 2002

[2] For instructions on making such figures with *Cabri Geometry II* and *Geometer's Sketchpad* see E3-2$\mathcal{DGS}$.

(c) Continue this reflection process, first reflecting in line ℓ and then in line m until you end up with a flag on top of F_0.

(d) Now select line ℓ and rotate it about P using the *rotate tool*.[3] (The angle $\angle(\ell, m)$ should remain the same during this rotation.)

(e) Replace your previous caption/comment with a new version that summarizes your observations. In particular, answer the following questions:

i. Which symmetries are exhibited by your "multi-flag" figure?

ii. How is the multi-flag figure generated by these reflections related to the figure generated previously by your rotations?

iii. Describe how the image of a flag under a rotation can also be obtained by using only reflections. Be sure to explain how many reflections are needed and describe the relation of the axes, i.e., mirror lines, to the center and angle of the rotation.

iv. When the location of line ℓ (and consequently of m) is changed without changing the intersection point or angle between these lines, which flag locations change? Which stay the same?

v. How many different pairs of lines ℓ and m can be used as axes of reflections so that the result of reflecting Flag F_0 in ℓ and then in m is the same as the result of rotation Flag F_0 around center P through angle θ?

(f) Save your final construction as "R&R-Flag."

Report:

Submit a computer folder titled "E3-3CG" or "E3-3GS" (depending on whether you used *Cabri* or *Sketchpad*). This folder should contain the two figures/sketches listed below and any other dynamic geometry software figures/sketches you made for the activities in section 3.3 of the text *A Course in Modern Geometries, 2nd. Ed.*[4]. For each, be sure to use the appropriate extension, i.e., ".gsp" for *Sketchpad* sketches and ".fig" for *Cabri* figures.

(a) Rot-Flag (b) R&R-Flag

[3] You may need to apply the rotate tool to the second point used to construct ℓ.

[4] For each of these others, include a caption describing the purpose of the construction.