E3-2$\mathcal{DGS}$: Exploring Line and Point Reflections[1]

By Judith N Cederberg

Introduction:

To observe the effect of transformations, it is useful to apply them to non-symmetric objects such as flags. These can easily be constructed using dynamic geometry software ($\mathcal{DGS}$). This document details directions for using *Cabri Geometry II* ($\mathcal{CG}$) and *Geometer's Sketchpad* ($\mathcal{GS}$)to create flags and to implement line and point reflections. It also includes activities requiring the use of these reflections.

Equipment and Materials Needed:

1. A computer (Macintosh or a PC running Windows) with access to either *Cabri Geometry II* or *Geometer's Sketchpad (Version 3 or 4)*, and a disk on which to save files.

2. Two pre-made figures/sketches: *T-grid* and *B-tables*.

3.2.1 Constructing polygons and/or flags:

$\mathcal{GS}$ Using the **Point Tool** create the vertices of a non-symmetric polygon or flag by clicking at various locations. Use the **Pointer** to select pairs of vertices to be joined and select the *Segment* command in the **Construct Menu**.[2] To fill the interior of the flag, i.e., polygon, select all the polygon vertices and under the **Construct Menu**, select *Polygon Interior*. The color of the interior can be changed by selecting *Color* under the **Display Menu** and clicking on the desired entry in the color chart that appears.

$\mathcal{CG}$ In the **Lines Toolbox**, select the *(Nonregular) Polygon Tool.* Click at each vertex of your non-symmetric polygon or flag, completing it by reclicking on the original vertex. To fill the interior of the polygon, under the **Display Toolbox**, choose *Fill* and select a color from the color chart that appears. Move your cursor to a side of the polygon until the message "This polygon" appears. When it does, click the mouse.

3.2.2 Implementing Reflections:

A. Line Reflections:

$\mathcal{GS}$ *Reflection of an object F in line m* ($R_m(F)$).

Using the **Segment/Line Tool**, construct and label a line m. With line m selected, choose *Mark Mirror* under the **Transform Menu**. Then select the object F and choose *Reflect* under the **Transform Menu**.

$\mathcal{CG}$ *Reflection of an object F in line m* ($R_m(F)$).

Select *Reflection* from the **Transform Toolbox** and move your cursor over the object F. If you receive the pointer message "Reflect this [object name]" and the appropriate name appears, click the mouse and then click on the axis of reflection. If you receive the message "Which object?" hold down the mouse button and select the appropriate name and then click on the axis of reflection.

B. Point Reflections:

$\mathcal{GS}$ *Reflection of an object F in point C* ($R_C(F)$).

---

[1]Designed to supplement Section 3.2 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC July 8, 2002

[2]If you want a point to be located on a segment, select the segment and under the **Construct Menu**, select *Point on Object.* In Version 4, you can use the **Edit** command *Merge* to move a point onto a segment.

Construct and select a point to be the center C of a point reflection and choose *Mark Center* under the **Transform Menu**. Then select object F and choose *Dilate* under the **Transform Menu**.[3] In the dilate box that appears, enter the value -1.00 for the New Scale Factor (Be sure that the Old Scale Factor is 1.00) and click on *OK*.

$\mathcal{CG}$ *Reflection of an object F in point C ($R_C(F)$).*

Select *Symmetry*[4] from the **Transform Toolbox** and move your cursor over object F. If you receive the pointer message "Reflect this object name" and the appropriate name appears, click the mouse and then click on the center of reflection. If you receive the message "Which object?" hold down the mouse button and select the appropriate name and then click on the center of reflection.

3.2.3 Using Reflections:

A. Tiling a Square:

1. Open the figure/sketch "T-grid[5]." Using *only* reflections in the 3 heavy black lines, fill the entire grid (or at least the upper-lefthand corner of the grid) with blue tiles.
2. Are there any other sets of 3 lines that could also serve as axes of reflections to fill the grid? Could you have gotten by with fewer lines of symmetry? Create a caption/comment giving and explaining your answers. Be sure to specify and label any other lines that you could have used.
3. Save your tiling as "T-tiled."

B. Finding Paths of Billiard Balls:

1. Open the figure/sketch "B-tables."
2. Use reflections to find a path that will take a billiard ball from B to H on each table by bouncing the ball off the number of walls indicated. In each case you should assume that the ball will travel in a straight line once it is hit and that it will bounce off the walls of the table with a reflection angle equal to the incidence angle. Thus to find the spot on the wall at which you should aim, you will need to reflect the target position in the wall. When you need to bounce the ball off more than one wall, you should proceed by first finding the last point on the wall at which to aim, then the next-to-the-last point on the wall, etc[6].
3. Show the path of the ball on each table, hiding any intermediate constructions used to find the path. Then save your figure/sketch as "B-Paths."

Report:

Submit a computer folder titled "E3-2CG" or "E3-2GS" (depending on whether you used *Cabri* or *Sketchpad*). This folder should contain any dynamic geometry software figures/sketches you made for the activities in section 3.2 of the text *A Course in Modern Geometries, 2nd. Ed.*[7] and the two figures/sketches listed below. For each, be sure to use the appropriate extension, i.e., ".gsp" for *Sketchpad* sketches and ".fig" for *Cabri* figures.

(a) T-tiled (b) B-paths

[3] *Geometer's Sketchpad* has no specific "point reflection;" these instructions indicate a way of using the "Dilate" transformation. A rotation through 180^o could also be used.

[4] *Cabri Geometry* uses the name *Symmetry* rather than *Point Reflection.*

[5] Created by James King, University of Washington

[6] Adapted from problems in Section 1.5 of Richard G. Brown's *Transformational Geometry*, Dale Seymour Publications, ©1973.

[7] For each of these others, include a caption/comment describing the purpose of the construction.