E1-$\mathcal{GS}$: Introducing The Geometer's Sketchpad[1]

By Judith N Cederberg

Introduction:

The Geometer's Sketchpad enables on screen construction of geometric figures as well as elementary transformations of these figures. As you carry out the constructions described below, note any questions you have about specific features of the program. If this is your first exposure to a Macintosh drawing program, you will find that the beginning exercises will take considerably longer for you than for those who have some familiarity with such programs. However, your initial patience and persistence will soon pay off and you too will soon be a "pro."

Equipment and Materials Needed:

1. A computer (Macintosh or a PC running Windows) that has access to Version 3 or 4 of the program *The Geometer's Sketchpad*[2] and a disk on which to save files.
2. A copy of *The Geometer's Sketchpad* $\succ\succ\succ$ *Quick Reference*. This document briefly describes each of the tool box and menu options available on *The Geometer's Sketchpad*. More detailed information is available in the *Geometer's Sketchpad Reference Manual*.

How To Begin:

You will need to open the *Sketchpad* program by *double clicking* on the appropriate icon, i.e., by using the mouse to move the cursor on top of the appropriate icon and then pushing the mouse button twice in rapid succession. This should bring up a screen entitled "Untitled 1." To clear the logo from the screen, simply click the mouse button once. As in other drawing programs, you carry out commands by moving the cursor around the screen using the mouse and clicking on items you wish to select[3]. If you wish to deselect an object, you need to move the cursor to an empty portion of the screen and click.

General Procedures:

- **Constructions:** To carry out a desired construction, you will need to select the appropriate tool from the tool box by clicking on it and then moving the cursor (whose shape may now have changed) to the appropriate location. By clicking in this location, the desired construction should appear. To perform any one of the operations listed under the menus, you will need to select the object(s) on which you want to perform the operation by moving the cursor to the object and clicking on the object. Then you must move the cursor to the menu and *drag* it down through the menu options while holding down the mouse button. When the cursor is on top of the desired option, release the button.
- **Moving Objects:** To move an object to a new location on the screen, you must select the object and then *drag* it to the desired location by holding down the mouse button while moving the cursor to the new location.
- **Erasing:** To undo or erase a construction or transformation just completed, merely choose **Undo** under the **Edit** menu. If you want to redo the previously undone operation, choose **Redo**. If you wish to delete a small number of objects, leaving the rest of the construction intact, select the objects to be deleted and hit the "Delete" key.

Exploration Activities:

With this quick introduction, you should be able to carry out the elementary Euclidean constructions described below. These activities illustrate a number of the features of *The Geometer's Sketchpad*.

[1] Designed to supplement Chapter 1 in *A Course in Modern Geometries, 2nd Ed.*; revised by JNC July 2, 2002

[2] Key Curriculum Press

[3] Version 3 users who wish to select multiple objects will need to press the shift key while clicking on each object to be selected.

1.1.1 Triangle Construction and Measurement:

1. Using the *point tool*, construct three noncollinear points.
2. Using the *text tool*, show the labels of these three points and, if necessary change them to A, B, and C.[4]
3. Select the *select arrow tool*, then select both points A and B. Then choose **Segment** under the **Construct** menu. Similarly construct segments BC and AC. Complete your triangle by showing the labels assigned to these three segments.
4. Using the *select tool*, select each segment in turn. When a segment is selected, use the **Measure** menu to find the length of each side of ΔABC.
5. Select individual vertices in turn using the *select and translate tool*; and, while holding down the mouse, drag the selected vertex until the screen recorded measurements of the lengths of the sides of ΔABC indicate that you have an equilateral triangle.
6. Select all 3 points, A, C and B (in order) and use the **Measure** menu to measure $\angle ACB$. Does this measurement verify that ΔABC is an equilateral triangle?
7. Drag point A until the screen recorded measurement indicates that ΔABC is an isosceles right triangle, with right angle at C. To verify that you actually do have an isosceles right triangle, have *Sketchpad* carry out the calculation of $(2 * (length(segmentAC))^2)^{(1/2)}$ as follows:
   (a) Choose **Calculate** under the **Measure** menu.
   (b) Enter the expression to be calculated using the calculator that appears. To enter "$length(segmentAC)$" merely click on this measurement where it is displayed on your construction.
   (c) Also be sure to use parentheses around the "1/2" in the exponent.]
8. Select the entire triangle.[5] With the triangle selected, place the cursor on any part of the triangle and use the *translate tool* to drag the triangle to a new position on the screen. Do the angle and segment measurements change as you do this?
9. Use the following procedure to save your figure on your disk:
   - Place your disk in the disk drive, if you haven't already done so.
   - Select **Save As** under the **File** menu. Then enter the name "Isos-rt-triangle," or if you are restricted to 8 characters, use "Is-rt-Tr. Note that some computers automatically add the ".gsp" extension. If yours does not, add the ".gsp" extension.
   - Click on **Drive** until the name of your disk appears. Then click on **Save**.

Comments:

As the previous exercises demonstrated, the triangle you constructed was non-rigid, i.e., the lengths of its sides and the measure of its angles changed if a vertex (or a side) was dragged to a new position. There are times when it is important to generate figures that maintain certain properties when either their vertices or sides are subjected to dragging. The following exercise is intended to lead you through the construction of several such figures.

[4] To change a label, select the *text tool* and use it to double click on the current label. A label dialog box will appear in which you can then type the desired label. To delete a label, click on the labeled object (not on the label itself).

[5] To do this efficiently, click in a blank area above and to the left of the triangle. Then while holding down the mouse button, drag the cursor down and to the right to create a rectangular area of selection surrounding the triangle. When the area of selection surrounds the triangle, release the mouse button.

1.1.2 Equilateral Triangles, Squares and Rectangles:

1. Equilateral Triangle Construction:

   (a) **Open** a new sketch (see **File** menu). Then construct and label two points A and B and construct segment AB.

   (b) Select both points A and B and then create a circle with center A passing through B by choosing **Circle By Center + Point** (see the **Construct** menu). Construct a second circle with center at B passing through A.

   (c) Find one of the points of intersection of these circles by pointing the *arrow tool* at the intersection and clicking the mouse. Label this point C.

   (d) Finish the triangle by constructing the segments AC and BC. Then **Hide** both circles (see the **Display** menu). You should then have only the triangle showing.

   (e) Label and measure each of the sides of your triangle to verify that it is an equilateral triangle.

   (f) Select and drag one of the vertices. Watch the screen recorded measurements to verify that the shape of the triangle remains the same as you increase or decrease the length of a side.

   (g) Save your figure as "Equ-triangle.gsp" or "EquTri.gsp"

2. Square Construction:

   (a) **Open** a new sketch (see **File** menu). Then construct and label two points A and B and construct segment AB.

   (b) Select point A and choose **Mark Center** (see the **Transform** menu). Now select segment AB and point B and choose **Rotate** (see the **Transform** menu) to rotate them to obtain a second side and third vertex of a square. Label this new vertex D.

   (c) Use another rotation to create a third side and a fourth vertex of the square. Label this final vertex C. Then construct the remaining side of the square.

   (d) Save your figure as "Square.gsp"

3. Rectangle Construction:

   (a) Open a new sketch and construct a segment AB and the perpendicular to the segment at B (see the **Construct** menu). Locate and label a point C on this perpendicular by selecting the perpendicular and choosing **Point On Object** (see the **Construct** menu). Now **Hide** line BC (see the **Display** menu) and construct segment BC.

   (b) Select segment AB and point C and construct the parallel to AB through C (see the **Construct** menu). Also construct a line through A parallel to BC. Then construct the point of intersection of these two lines (see **Construct** menu). Label this point D. Finally, hide lines AD and CD and construct segments AD and DC.

   (c) Measure each of the segments and one of the angles to verify that you have a rectangle.

   (d) Check that quadrilateral ABCD remains a rectangle as you drag its vertices and edges, and verify that you can change the length of any one side without changing the length of the adjacent sides. Why do you get different effects by dragging different vertices?

   (e) Select the *text tool*, move the cursor to an empty region of your sketch and double click to create a caption box. In the caption box type out a description that explains how to change the various dimensions of your rectangle[6].

   (f) Save your figure as "Rectangle.gsp" or "Rectang.gsp"

[6] You can resize the caption box by clicking on it with the *select tool* and then dragging one of the handles that appear.

1.1.3 Creating Custom Tools/Scripts for Regular n-gons:

Once you have carried out a construction, *Geometer's Sketchpad* enables you to create a *tool*[7] that will enable you to duplicate the construction by merely setting up the initial geometric objects required for the construction and then applying the tool. The following exercise will direct you in the creation of a tool for an equilateral triangle. (Be sure to follow the directions appropriate for your version of *Sketchpad*.) You are then asked to make tools for a square and a regular hexagon.

1. An Equilateral Triangle Tool:

   Version 4 Directions:

   (a) In a new sketch, construct two points. Then construct an equilateral triangle with these two points as vertices.
   (b) Select the 3 vertices and 3 sides of your triangle but no other objects.
   (c) Select the *Custom Tool* by clicking on the bottom icon in the side toolbar.
   (d) Choose *Create New Tool* in the window that appears and name your tool "Equilateral Triangle." Your tool will be saved as part of the current figure[8].
   (e) Select the *Show Script View* option to see the script that has been recorded for your tool.
   (f) Save your figure (and tool) as "Equ-Tool.gsp."
   (g) Try out your tool by selecting your tool from the *Tool Box* and then clicking at two positions that will be vertices of your triangle.

   Version 3 Directions:

   (a) Open a new sketch and a new script. You will now have two windows open on the screen. The sketch window will contain your construction and the script window will contain a record of the steps used in your construction. If at any point you wish to activate the window in which you are not currently working, simply move the cursor to that window and click on it. If you want to resize one of the windows, click on the size box in the lower right corner of the window and drag the box to the position desired. An entire window can be moved by clicking on the title bar at the top of the window and dragging the window.
   (b) Before recording a script for a construction it is a good idea to think through the process you will use in your construction. In this case you may wish to review the process you used to construct an equilateral triangle in the previous section.
   (c) When you are ready to begin your construction, "push", i.e. choose **Rec** on the script window. Then move back to the sketch window and carry out the construction as outlined above. As soon as you are finished, "push" **Stop** on the script window. Now save the script as "Equ-tri.gss." The ".gss" extension identifies this as a script rather than a sketch.
   (d) To try out the script, open a new sketch and construct objects corresponding to the given objects listed in your script (in this case, the givens should be two points). Select these objects and push **Play** on the tape recorder.
   (e) Try out the script in another sketch but this time experiment with the **Step** and **Fast** options.

2. Create a tool/script for a square. Save it in Version 4 as "Squ-Tool.gsp" or in Version 3 as "Square.gss"

3. Create a tool/script for a regular hexagon. Save it in Version 4 as "Hex-tool.gsp" or in Version 3 as "Reg-hex.gss"

Report: Submit a computer folder titled "E1-GS" containing:

Sketches:	*Version 4 Sketches & Tools:*	*Version 3 Scripts*
(a) Isos-rt-triangle.gsp	(a) Equ-Tool.gsp	(a) Equ-tri.gss
(b) Equ-triangle.gsp	(b) Squ-Tool.gsp	(b) (b) Square.gss
(c) Square.gsp	(c) Hex-tool.gsp	(c) Reg-hex.gss
(d) Rectangle.gsp		

[7] Version 3 refers to this as a *script*.

[8] To find directions for creating a special Tool Folder in which to store your Custom Tools, see the Sketchpad Help menu.