The set of integers

Let $\mathbb{Z}$ denote the set of integers, i.e.,

$$\mathbb{Z} = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}.$$

We write

- $3 \in \mathbb{Z}$ to say that 3 is an element of $\mathbb{Z}$.
- $\frac{2}{5} \notin \mathbb{Z}$ to say that $\frac{2}{5}$ is *not* an element of $\mathbb{Z}$.

Discuss in your group: Here is a subset of $\mathbb{Z}$,

$$A = \{3, 5, 7, \dots, 17, 19\}.$$

Write down all of its elements. How many are there?

What's in this subset?

Question: Write down all the elements of $A = \{3, 5, 7, \ldots, 17, 19\}$.

??

Answer: $A = \{3, 5, 7, 11, 13, 17, 19\}$

Key: Rather than listing elements, describe the *property* they satisfy.

$$A = \{n \in \mathbb{Z} \mid 3 \leq n \leq 19 \text{ and } n \text{ is prime}\}.$$

where the elements of set A come from.

"such that" (inside { } only.)

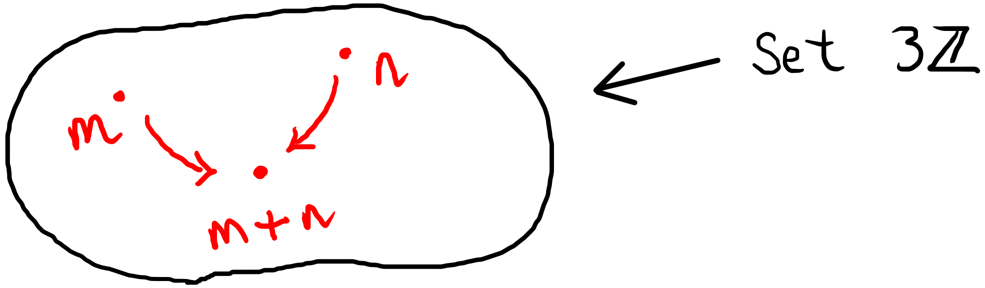
property satisfied by elements of set A.

Closed under addition

Let $3\mathbb{Z} = \{n \in \mathbb{Z} \mid n = 3k \text{ where } k \in \mathbb{Z}\}$, i.e., the set of multiples of 3.

Prove: If $m, n \in 3\mathbb{Z}$, then $m+n \in 3\mathbb{Z}$. (So, the set $3\mathbb{Z}$ is closed under addition.)

Picture:



Proof: Assume $m, n \in 3\mathbb{Z}$.

Then $m = 3k$ and $n = 3j$ where $k, j \in \mathbb{Z}$.

Thus, $m + n = 3(k+j) = 3k + 3j \in 3\mathbb{Z}$, since $k + j \in \mathbb{Z}$.

swap

Hence, $m + n \in 3\mathbb{Z}$.

Proving set equality

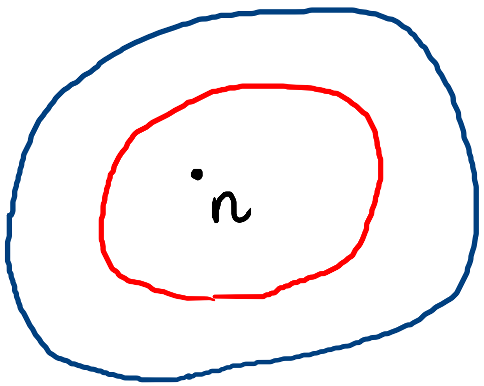
Let $C = \{n \in \mathbb{Z} \mid n \in 2\mathbb{Z} \text{ and } n \in 3\mathbb{Z}\}$, i.e., the set of multiples of 2 *and* 3.

Claim: $C = 6\mathbb{Z}$, i.e., the sets C and $6\mathbb{Z}$ are equal. (How can we *prove* it?)

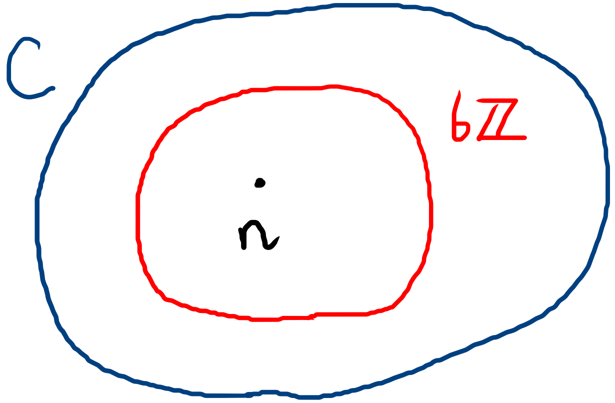
Proof know-how: To show that sets S and T are equal...

- We must prove *two* things:
  - $S \subseteq T$ (i.e., S is a subset of T)
  - $T \subseteq S$ (i.e., T is a subset of S)
- How do we show that, say, $S \subseteq T$?
  - $S \subseteq T$ means every element of S is an element of T.
  - Written as an implication: If $n \in S$, then $n \in T$.
  - Assume $n \in S$. Then show that $n \in T$.

$S \subseteq T$



Problem #5(a)



Claim: $6\mathbb{Z} \subseteq C$. (See HW for $C \subseteq 6\mathbb{Z}$.)

Recall: $C = \{n \in \mathbb{Z} \mid n \in 2\mathbb{Z} \text{ and } n \in 3\mathbb{Z}\}$.

Proof: We must prove... If $n \in 6\mathbb{Z}$, then $n \in C$.

Assume $n \in 6\mathbb{Z}$.

Then $n = 6k$ for some integer k.

Thus, $n = 2 \cdot 3k$ where $3k \in \mathbb{Z}$. Therefore, $n \in 2\mathbb{Z}$.

Similarly, $n = 3 \cdot 2k$ where $2k \in \mathbb{Z}$. So, $n \in 3\mathbb{Z}$.

Hence, $n \in C$.