

Abstract Algebra
Day 1 Class Work

1. Which of these are true?

- (a) If I live in Tokyo, then I live in Japan.
- (b) If I live in Japan, then I live in Tokyo.
- (c) If I don't live in Tokyo, then I don't live in Japan.
- (d) If I don't live in Japan, then I don't live in Tokyo.

Hint: I grew up in Osaka.

2. **Prove:** If n is an integer, then $n^2 + n$ is even.

Hint: What if n is odd?
What if n is even?

3. **Prove:** If n^2 is odd, then n is odd. (Here, n is an integer.)

Hint: Setting $n^2 = 2k + 1$ is *not* a good idea. (Why not?)

4. Consider the statement:

If n is an odd integer, then n is a prime number.

Which value of n is a *counterexample* showing that the statement is false? Explain.

- $n = 13$
- $n = 15$

← Just one counterexample is needed to show that a statement is false.

5. Consider the statement:

If n is a prime number, then $n + 2$ is also a prime number.

Find a counterexample to show that the statement is false.

6. Consider the following statements:

- If n is prime, then $2^n - 1$ is prime.
- If $2^n - 1$ is prime, then n is prime.

Determine whether or not each statement is true. If it's true, prove it. If it's false, come up with a counterexample.

Hint: Start by trying out some concrete values of n .