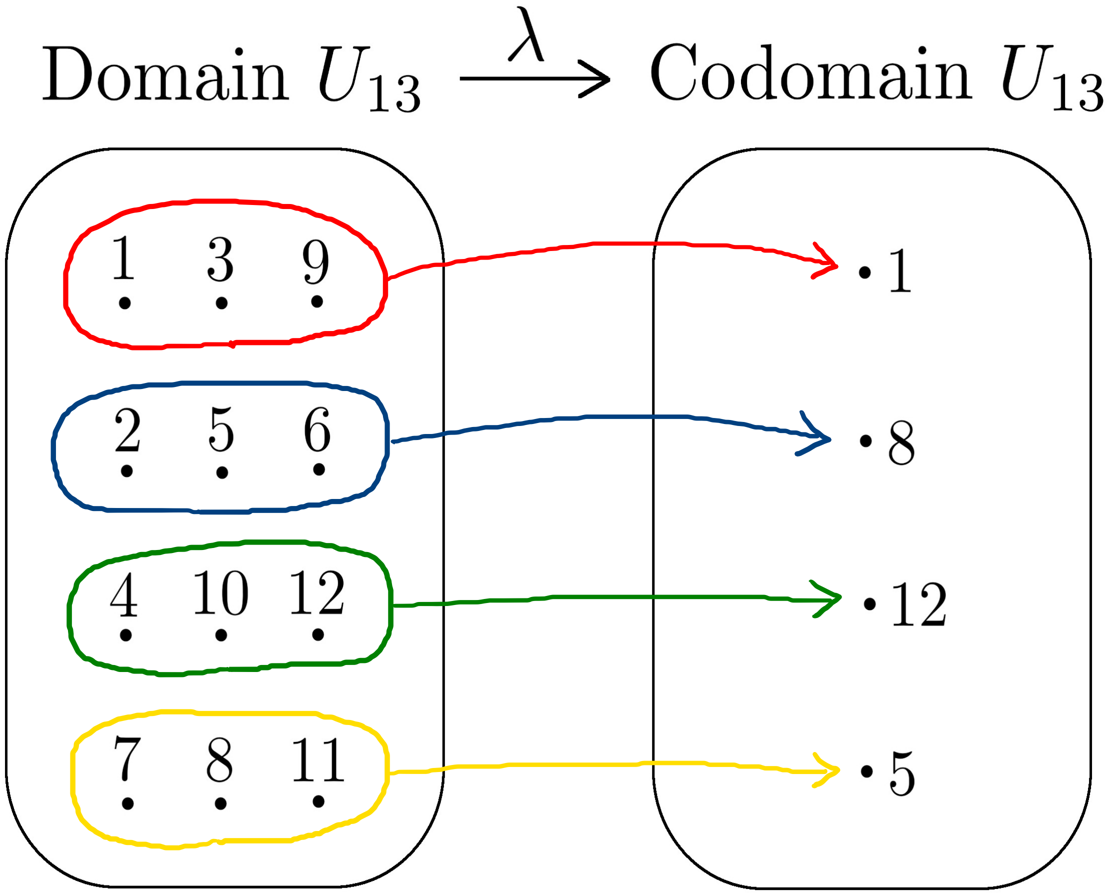
Consider $\lambda : U_{13} \to U_{13}$ where $\lambda(a) = a^3$ for all $a \in U_{13}$.

$1^3 = 1$
$2^3 = 8$
$3^3 = 1$
$4^3 = 12$
$5^3 = 8$
$6^3 = 8$
$7^3 = 5$
$8^3 = 5$
$9^3 = 1$
$10^3 = 12$
$11^3 = 5$
$12^3 = 12$

(Not all elements are shown.)

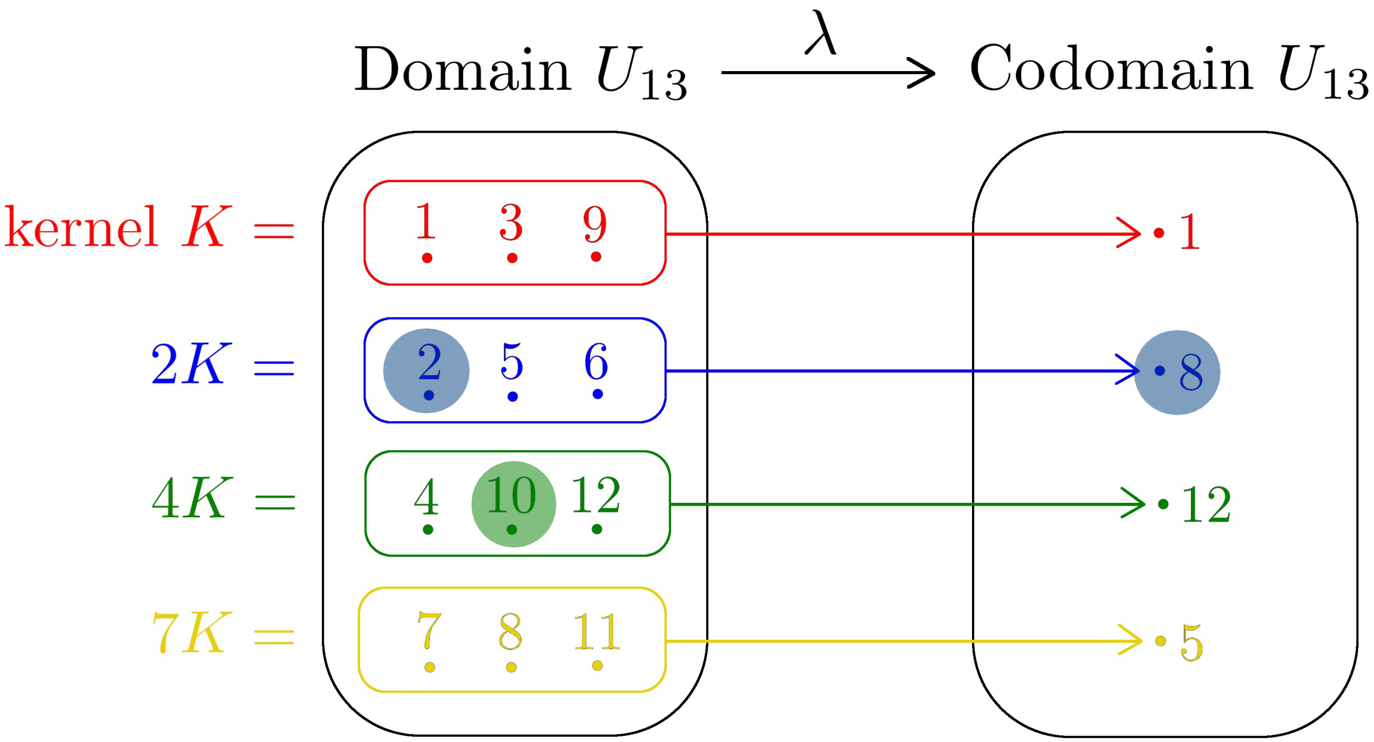
$\ker\lambda =$



$\operatorname{im}\lambda = \{1,\ 8,\ 12,\ 5\}$

Observation: A group homomorphism partitions the domain into **equal-sized** subsets. (Why? We'll see *very* soon!)

Consider $\lambda : U_{13} \to U_{13}$ where $\lambda(a) = a^3$ for all $a \in U_{13}$.



Newsflash:

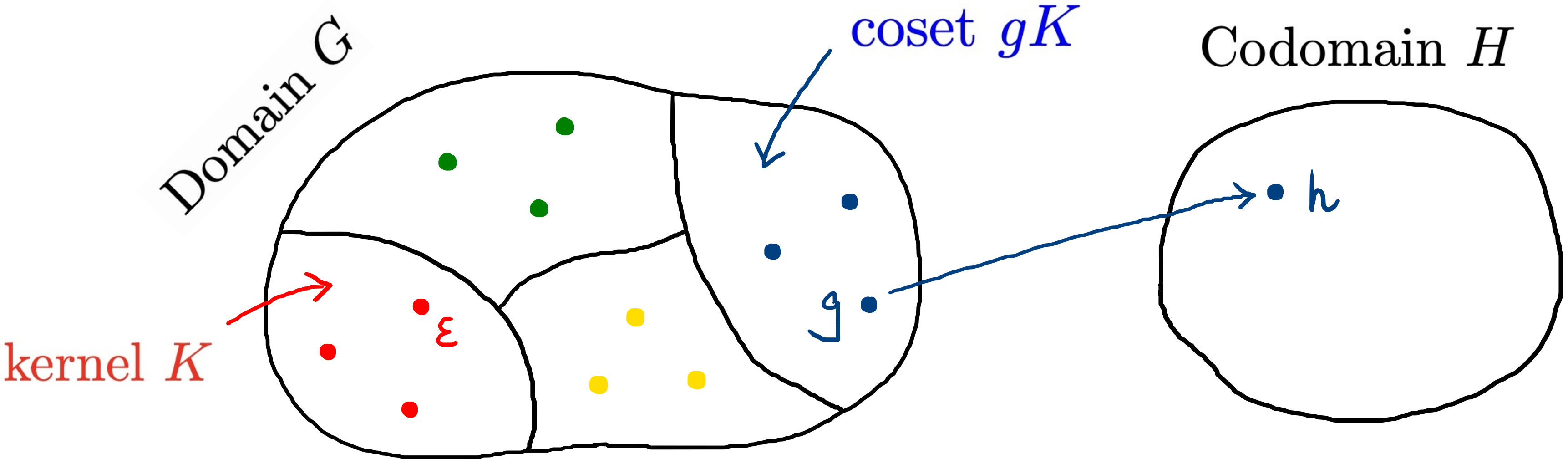
The cosets of $K = \ker \lambda$ partition the domain U_{13} in the same way λ does.

- For $2 \in U_{13}$ (domain), we have $\lambda(2) = 2^3 = 8$.
- Every element of coset $2K$ also map to 8. $\leftarrow$ If $a \in 2K$, then $\lambda(a) = 8$.
- In fact, only the elements of $2K$ map to 8. $\leftarrow$ If $a \notin 2K$, then $\lambda(a) \neq 8$.

Theorem: Let $\theta : G \to H$ be a group homomorphism with $K = \ker\theta$. Let $g \in G$ such that $\theta(g) = h$ where $h \in H$. Given $a \in G$, we have...

(1) If $a \in gK$, then $\theta(a) = h$ (i.e., every element of coset gK maps to h).

(2) If $a \notin gK$, then $\theta(a) \neq h$ (i.e., *only* the elements of gK map to h).



Conclusion:

- The cosets of K partition the domain in the same way that θ does.
- Hence, the subsets created by the homomorphism θ are *equal-sized*.

Theorem: Let $\theta : G \to H$ be a group homomorphism with $K = \ker\theta$. Let $g \in G$ such that $\theta(g) = h$ where $h \in H$. Given $a \in G$, we have...

(2) If $a \notin gK$, then $\theta(a) \neq h$.

Proof. We will prove: if $\theta(a) = h$, then $a \in gK$.

Assume $\theta(a) = h$. Moreover, $\theta(g) = h$.

Hence, $\theta(g^{-1} \cdot a) = \theta(g)^{-1} \cdot \theta(a) = h^{-1} \cdot h = \varepsilon_H$.

Thus, $g^{-1} \cdot a \in K$.

Let $g^{-1} \cdot a = k$ for some $k \in K$.

Then $a = gk \in gK$.

Thus, $a \in gK$.

Scrap:

$\theta(a) = h$, $\theta(g) = h$.

* Goal: Show

$\theta(g^{-1} \cdot a) = \varepsilon_H$.

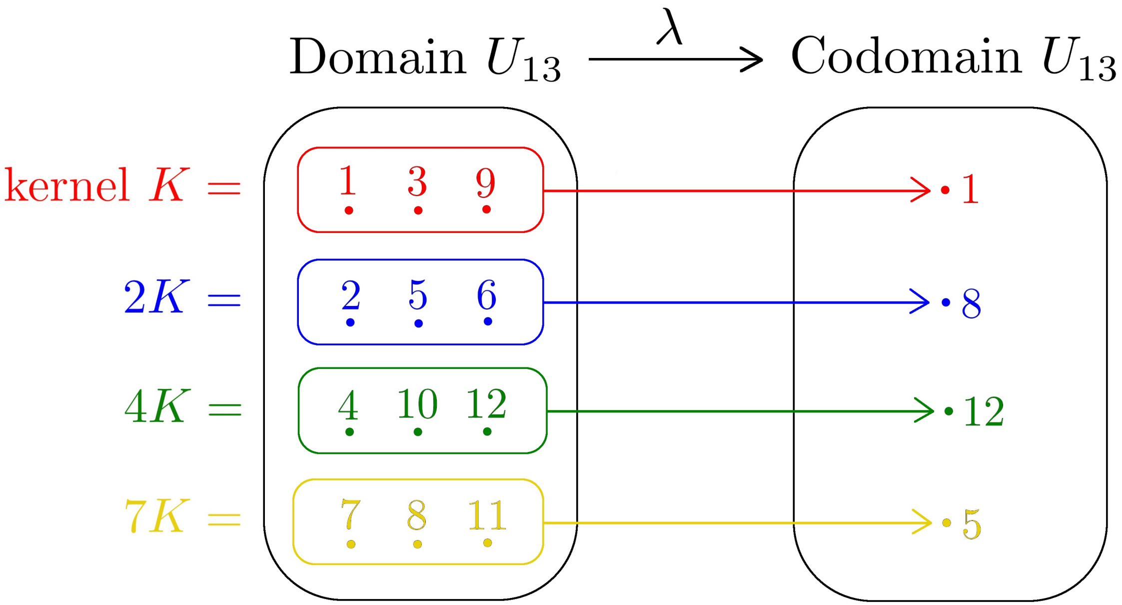
$\theta(g)^{-1} \cdot \theta(a) = h^{-1} \cdot h = \varepsilon_H$.

$g^{-1} \cdot a = k$

$a = gk$ $(k \in K)$

$a \in gK$

Consider $\lambda : U_{13} \to U_{13}$ where $\lambda(a) = a^3$ for all $a \in U_{13}$.



Crazy idea: Compare...

- $U_{13}/K = \{1K, 2K, 4K, 7K\}$
- $\operatorname{im} \lambda = \{1, 8, 12, 5\}$

$\cdot$	$1K$	$2K$	$4K$	$7K$
$1K$	$1K$	$2K$	$4K$	$7K$
$2K$	$2K$	$4K$	$7K$	$1K$
$4K$	$4K$	$7K$	$1K$	$2K$
$7K$	$7K$	$1K$	$2K$	$4K$

$\cdot$	1	8	12	5
1	1	8	12	5
8	8	12	5	1
12	12	5	1	8
5	5	1	8	12

First Isomorphism Theorem.

Let $\theta : G \to H$ be a group homomorphism with $K = \ker\theta$.

Then $G/K \cong \operatorname{im}\theta$, where $gK \in G/K$ corresponds to $\theta(g) \in \operatorname{im}\theta$.