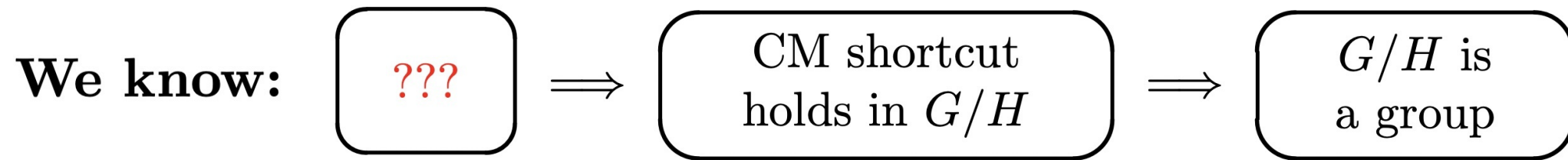


Let G be a (multiplicative) group and H a subgroup.

Coset multiplication shortcut: For $aH, bH \in G/H$, $aH \cdot bH = (ab)H$.

IF the shortcut holds in G/H , then...

G/H is a **quotient group** under coset multiplication.



- We'll explore $???$ next time.
- For today, assume that the shortcut holds.

Let G be a group and H a subgroup. Suppose that G/H satisfies the coset multiplication shortcut.

Discuss in your group: Explain why

Last time: $(aH)^{-1} = a^{-1}H$.

$$(aH)^3 = a^3H \text{ and } (aH)^{-3} = a^{-3}H \text{ for all } aH \in G/H.$$

$$\bullet (aH)^3 = aH \cdot aH \cdot aH \stackrel{CM}{=} a^3H.$$

$$\begin{aligned} \bullet (aH)^{-3} &= ((aH)^{-1})^3 = (aH)^{-1} \cdot (aH)^{-1} \cdot (aH)^{-1} \\ &= a^{-1}H \cdot a^{-1}H \cdot a^{-1}H \stackrel{CM}{=} (a^{-1})^3H = a^{-3}H. \end{aligned}$$

Conclusion: $(aH)^k = a^kH$ for all $k \in \mathbb{Z}$.

Theorem: Suppose $[G : H] = n$. Then $g^n \in H$ for all $g \in G$.

Proof: Let $g \in G$. We must show that $g^n \in H$.

We know that $n = [G : H] = \text{size of } G/H$.

Consider $gH \in G/H$, and let $d = \text{ord}(gH)$ in G/H .

Then $d \mid n$ so that $n = d \cdot k$ for some integer k .

Thus, $(gH)^n = (gH)^{dk} = ((gH)^d)^k = (\varepsilon H)^k = \varepsilon H$,

so that $(gH)^n = \varepsilon H$.

Also, $(gH)^n = g^n H$, and so $g^n H = \varepsilon H = H$.

Therefore, $g^n \in H$.

Scrap: Let $g \in G$.

* Work with

$gH \in G/H$

* $n = [G : H]$
= size of G/H

$\Rightarrow (gH)^n = \varepsilon H$

$\Rightarrow g^n H = H$

$\Rightarrow g^n \in H$.