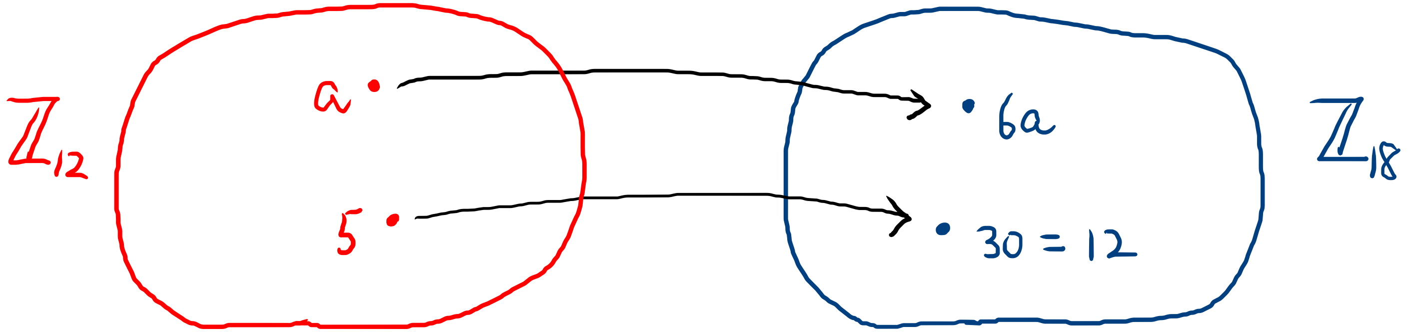
Consider the function $\gamma : \mathbb{Z}_{12} \to \mathbb{Z}_{18}$ where $\gamma(a) = 6a$ for all $a \in \mathbb{Z}_{12}$.

Example:

$$\begin{aligned}\gamma(5) &= 6 \cdot 5 \\ &= 30 = 12\end{aligned}$$

in $\mathbb{Z}_{18}$



- The *domain* of the function is $\mathbb{Z}_{12}$.
  This is the set of all possible inputs into the function.

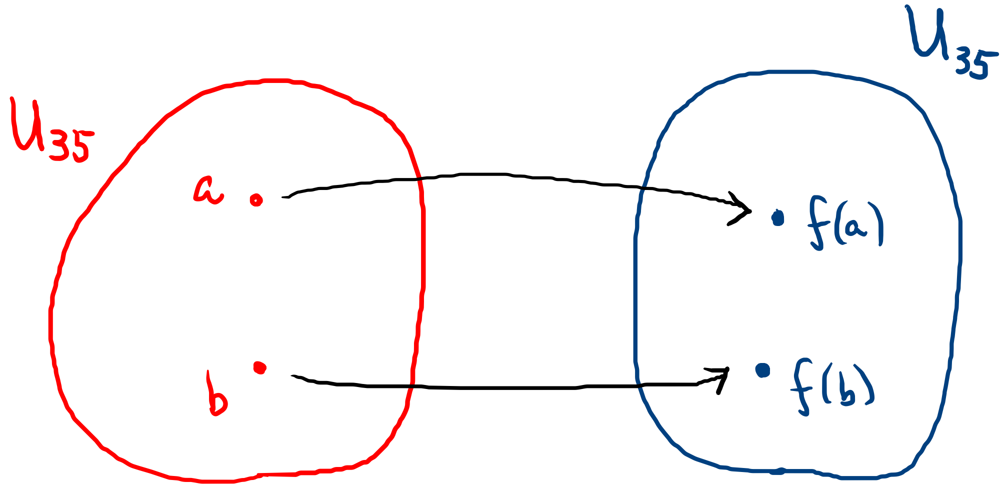
- The *codomain* of the function is $\mathbb{Z}_{18}$.
  This set contains all outputs (and possibly other elements).

- The *rule* of the function is $\gamma(a) = 6a$,
  where $a \in \mathbb{Z}_{12}$ (domain) and $\gamma(a) \in \mathbb{Z}_{18}$ (codomain).

One-to-one (example)

Consider $f : U_{35} \to U_{35}$ where $f(x) = 3x$ for all $x \in U_{35}$.

Claim. If $a \neq b$, then $f(a) \neq f(b)$.



Proof. We'll prove the contrapositive:

$$\text{If } \underline{f(a) = f(b)}, \text{ then } \underline{a = b}.$$

Assume $f(a) = f(b)$, so that $3a = 3b$.

Noting that $3^{-1} = 12$, we multiply both sides by 12.

Hence, $12(3a) = 12(3b) \implies (12 \cdot 3)a = (12 \cdot 3)b \implies 1 \cdot a = 1 \cdot b$.

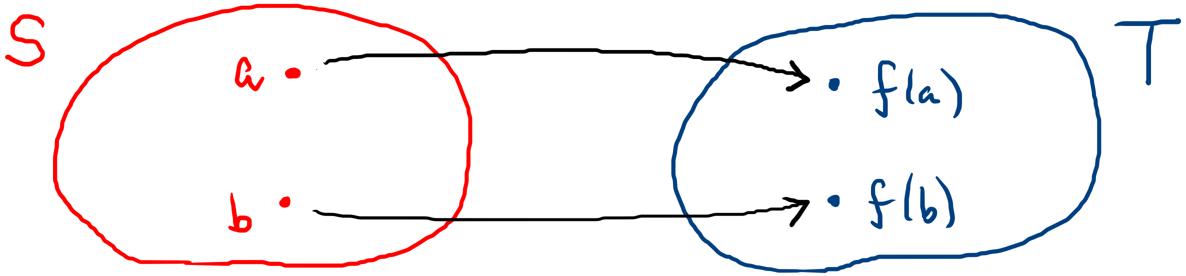
Thus, $a = b$.

One-to-one (general)

Let $f : S \to T$ be a function from domain S to codomain T.

Definition. f is said to be **one-to-one** when it satisfies:

$$a \neq b \implies f(a) \neq f(b) \quad \text{for all } a, b \in S.$$



Key:
Different inputs map to different outputs.

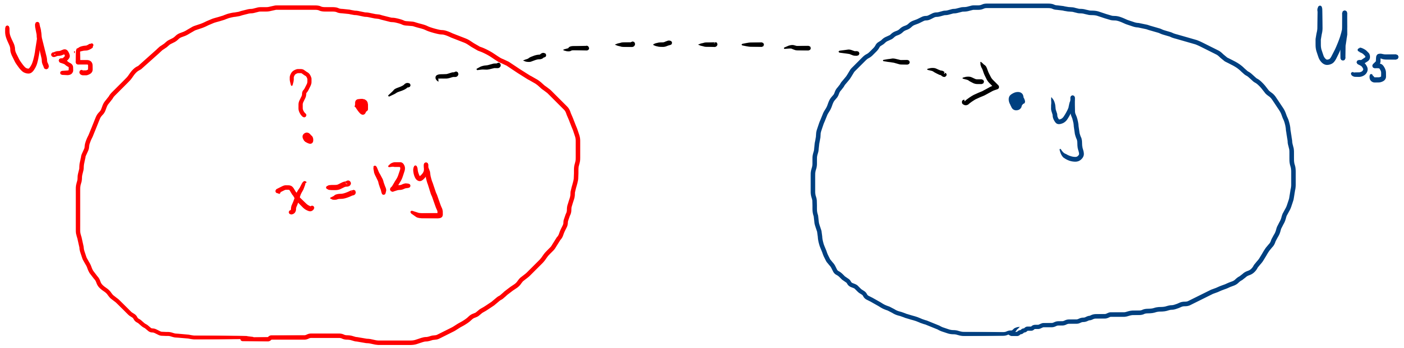
Proof know-how: To show that f is one-to-one. . .

- Assume $f(a) = f(b)$, where $a, b \in S$.
- Show that $a = b$.

i.e., prove the contrapositive.

Onto (example): Consider $f : U_{35} \to U_{35}$ where $f(x) = 3x$ for all $x \in U_{35}$.

Claim: If $y \in U_{35}$, then there exists $x \in U_{35}$ such that $f(x) = y$.



Key: $y \in U_{35}$ in the codomain is chosen first.

Proof: Assume $y \in U_{35}$. We must find $x \in U_{35}$ such that $f(x) = y$.

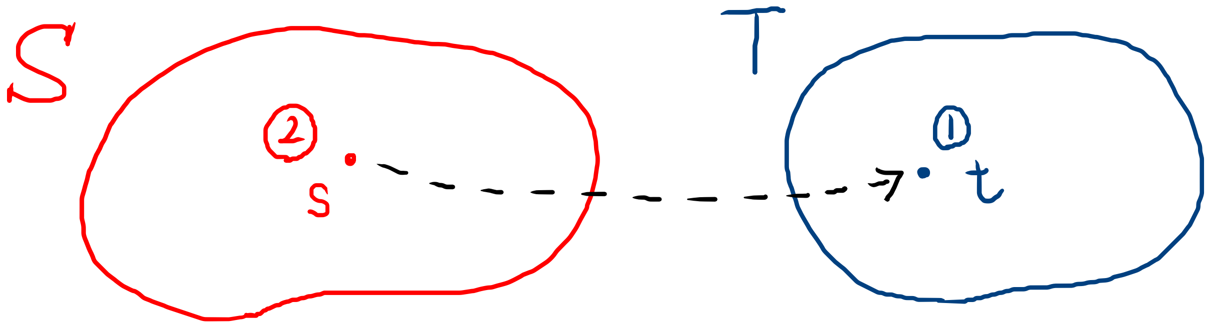
Noting that $3^{-1} = 12$, let $x = 12y$.

We know that $x \in U_{35}$, because $12, y \in U_{35}$ and U_{35} is closed.

Then, $f(x) = 3(12y) = (3 \cdot 12)y = y$, as desired.

Onto (general): Consider $f : S \to T$ with domain S and codomain T.

Definition. f is **onto** if for every $t \in T$, there exists $s \in S$ such that $f(s) = t$.



> **Key:** Every element of the codomain gets "hit" by f.

Proof know-how: To show that f is onto...

- Let $t \in T$. (Example: Let $y \in U_{35}$.)

$(x = 12y \in U_{35}.)$

- Find $s \in S$ such that $f(s) = t$. This s is usually expressed in terms of t.
- **(If necessary)** Verify that s is actually in set S.
- Show that s satisfies the desired property, namely $f(s) = t$.