Warm-up problem

Claim: *If n is an odd integer, then n^2 is odd.*

Discuss in your group:

(a) Create a few concrete examples to convince yourselves that the claim is true.

Example: $n = 7$ is odd $\implies n^2 = 49$, which is also odd.

Remark: Concrete examples are *really* important.

(b) Write a clear, precise, convincing, and [insert superlative] argument explaining **why** the claim is true.

Odd and Even

Examples:

- 7 is odd, because $7 = 2 \cdot 3 + 1$
- 213 is odd, because $213 = 2 \cdot 106 + 1$
- -1081 is odd, because $-1081 = 2 \cdot (-541) + 1$

Definitions:

- An integer n is *odd* when $n = 2 \cdot k + 1$ for some integer k.
- An integer n is *even* when $n = 2 \cdot k$ for some integer k.

Implication (i.e., an "if ..., then ..." statement)

Claim: If n is an odd integer, then n^2 is odd.

(hypothesis) (conclusion)

Proof know-how: To *prove* an implication...

✓ 1. *Assume* that its hypothesis is true.

2. *Show* that its conclusion is true.

Proof: Assume that n is an odd integer.

Then $n = 2k+1$ for some integer k.

Hence $n^2 = (2k+1)(2k+1) = 4k^2 + 4k + 1 = 2(2k^2 + 2k) + 1$,

where $2k^2 + 2k$ is an integer.

Thus, n^2 is odd.

Problems #2 and #3

2. **Prove:** If n is an integer, then $n^2 + n$ is even.

   Proof: Use proof by cases. (See reading.)

3. **Prove:** If n^2 is odd, then n is odd. (Here, n is an integer.)

   Proof: Assume that n^2 is odd.

   Then $n^2 = 2K + 1$ for some integer K.

   Hence $n = \sqrt{2K+1}$ ... ?!

   Thus, n is odd.

Problem #1: Which of these are true?

(a) If I live in Tokyo, then I live in Japan. T

(b) If I live in Japan, then I live in Tokyo. F

(c) If I don't live in Tokyo, then I don't live in Japan. F

(d) If I don't live in Japan, then I don't live in Tokyo. T

For simplicity...

- Let T be "I live in Tokyo."
- Let J be "I live in Japan."

$\Longrightarrow$

Then...

- (a) says: If T, then J.
- (d) says: If not J, then not T.

So what? Well...

- Implications (a) and (d) are contrapositives of each other. (Ditto for (b) and (c).)

Key: Contrapositives are *equivalent*.

Contrapositive

- We had to prove: If n^2 is odd, then n is odd.
- But it's hard to prove this implication directly.
- Instead, we can prove its contrapositive, namely. . .

even even

* If n is *not* odd, then n^2 is *not* odd.

Proof know-how:

Proving "If p, then q" is the same as proving the contrapositive "If not q, then not p."