Undergraduate Research

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Student Involvement. Algebraic combinatorics, as a field of research, is naturally built for the inclusion of students at many levels of education not only in mathematics but in physics and computer science as well. The elevator pitch that I give for my work explains how algebraic combinatorics is just a term used for taking complex mathematical structures and turning them into puzzles; if we can solve the puzzle, then we have solved the underlying math. By presenting the puzzle to a student as a purely combinatorial object, it can be quickly understood.

Making Mathematics Available to a Greater Audience. My views concerning the inclusion of students in research are similar to those I hold in my classrooms: everyone learns differently and a main role that we hold as educators is to help students excel by offering a multitude of modalities to match their individual strengths. Our students come from diverse backgrounds with varying mathematical experiences. It is our job to meet them at their level and provide them with the opportunities needed to grow.

- For students who are algorithmic learners, I can present research in this field through the use of coding.
- For those who are visual learners, we can start with the fillings of Young diagrams.
- For students who are spatial learners, we can start with reflections in euclidean space and build up the field through root systems.

Once a student has an understanding of the combinatorial models, building them through one of the above mentioned modalities or through other means, we can start to explore certain statistics as well as maps between other combinatorial models. Research in mathematics is often limited to students who excel in certain proof writing courses. **My hope is that we can take advantage of the very nature of this field to offer opportunities to a greater student audience.** We can show students that their strengths can be math strengths, even if they are not traditionally deemed as such.

Recent Student Projects.

I: During the spring 2022 semester, I worked with one of my Calculus II students. We extended the crystal bijections given in [1] to explicit crystal isomorphisms. The student's part was to show that the *Extend* algorithm given in the associated bijection is invariant under the crystal operators. These can be described through the associated combinatorics with no algebraic background so the project focuses on being an introduction to proof writing and logical thinking. We also explored a bit of LaTeX and started an introduction to Python and Sage.

II: Last summer, I was able to fund three students for ten weeks of full time research. Our project extended the work of [4] to other classical types in a special case. This project required Linear Algebra as background and provided upper level students with a more rigorous problem. The students learned the underlying algebra and how to write software to run tests. We used those tests to propose and prove results. The goal of this project was to introduce the students to the whole journey of mathematical research in this field.

Upcoming Student Projects. I have funding for 3 students to each do 50 hours of research during the upcoming Spring 2023 semester and they will be working on the following projects.

III: Two students will be working on a project where we will be finding the preimages of sorting algorithms. This has recently been done for the classic sorting algorithms. We will be doing similar work but with the four recently discovered "Shuffle Sort" algorithms. This project will again demonstrate the interface of python coding and mathematical research as well as develop LaTeX skills as we write up our results.

IV: One student will be working on a project with Parking Functions. We will be following the work of Harris and her students in their paper *Parking Functions: choose your own adventure*. This will be available to a student with less mathematical experience. The main goal of the project will be to learn how to create a math problem to study and to begin to learn how to play with that problem and then prove your results.

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